

**METR 4433 – Mesoscale Meteorology  
Spring 2017**

**Problem Set #3  
Atmospheric Instability**

Distributed Saturday, 25 February 2017  
Due Tuesday, 21 March 2017

**INSTRUCTIONS:** Please answer each of the questions shown below. Pay close attention to neatness and describe your work at each step of the solution process.

1. In class, we derived the following equation governing the vertical displacement of an air parcel from an initial location  $z = 0$ :

$$\frac{d^2z}{dt^2} = \frac{g}{T_o}(\gamma - \Gamma_d)z$$

We also showed that, for a stable atmosphere in which  $\gamma < \Gamma_d$ , the general solution is

$$z(t) = C_1 \cos(t\sqrt{A}) + C_2 \sin(t\sqrt{A})$$

Where  $A = \frac{g}{T_o}(\gamma - \Gamma_d)$  and  $C_1$  and  $C_2$  are constants.

(a). Show that  $\frac{g}{T_o}(\Gamma_d - \gamma) = \frac{g}{\theta_o} \frac{d\bar{\theta}}{dz} = B^2$

(b). Compute the period of oscillation of a parcel for a few environmental lapse rates,  $\gamma$ , that you choose on your own and comment on the results in light of the expression you derived in part a. How does the oscillation change as a function of the difference in lapse rates, and how does this relate to B?

2. (a) Consider a ring of air, at 30N latitude, having a radius of 300 km with no tangential velocity. If this ring is contracted to a radius of 50 km radius and moves to 35N latitude, find the resulting tangential velocity. Assume angular momentum,

$M = \frac{fr^2}{2} + Vr$ , is conserved. (b) Is your answer reasonable? EXPLAIN.

3. The diagram below shows a north-south vertical cross section of environmental isentropic (dashed) and absolute linear geostrophic momentum (solid) surfaces. As in class, assume uniformity in the x-direction.

Describe what happens to a tube of air, infinitely long and oriented in-and-out of the page, if it is displaced from location A to location B, from location C to location D, and from location E to location F, that is, three different cases, each involving the displacement of a tube along a straight line. EXPLAIN your reasoning for each case and note the signs of the slopes of the lines shown.

