1. Storm-Relative Environmental Helicity. The storm-relative environment helicity in the lowest 3 km layer is given as

$$SREH = \int_{0km}^{3km} [(\vec{V} - \vec{C}) \cdot \vec{\omega}_H] dz$$

which, based on the definition of the horizontal vorticity vector

$$\vec{\omega}_H = \hat{k} \times \frac{d\vec{V}}{dz} = -\frac{dv}{dz}\hat{i} + \frac{du}{dz}\hat{j},$$

can be rewritten as

$$SREH = -\int_{0km}^{3km} \hat{k} \cdot \left[(\vec{V} - \vec{C}) \times \frac{d\vec{V}}{dz} \right] dz = -\int_{0km}^{3km} \hat{k} \cdot \left[(\vec{V} - \vec{C}) \times d\vec{V} \right] = -\int_{0km}^{3km} \hat{k} \cdot \left[\vec{V}_r \times d\vec{V} \right]$$

where $\vec{V}_r \equiv (\vec{V} - \vec{C})$ is the storm-relative velocity.

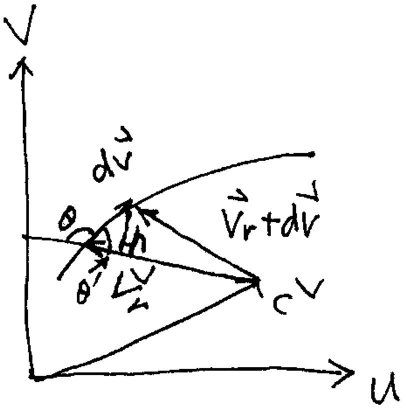
a). Using the above information and your knowledge of analytic geometry, show that the SREH is equal to minus twice the signed (i.e., positive or negative) area swept out by the storm-relative wind vector between 0 and 3 km on a hodograph. Note that, by convention, an area is positive (negative) if it is swept out counterclockwise (clockwise). To keep the problem simple, assume that wind observations are available at the 0 and 3 km levels only.

There is more than one way for showing this. The following is only one example.

Consider the SERH in a layer of depth dz in which the wind vector increases from $\vec{V}_r$ to $\vec{V}_r + d\vec{V}$, the SERH in the layer is

$$d(SREH) = -\hat{k} \cdot \left[\vec{V}_r \times d\vec{V} \right]$$

The total SERH in the 3 km layer will be sum of the SERH in each of such layers.



In the above Figure, we can see that

$$
\begin{aligned}
d(SREH) &= -\hat{k}\cdot\left[\vec{V}_r \times d\vec{V}\right] \\
&= -\hat{k}\cdot(-\hat{k})\left|\vec{V}_r\right|\left|d\vec{V}\right|\sin\theta = \left|\vec{V}_r\right|\left|d\vec{V}\right|\sin(\pi-\theta) \\
&= \left|\vec{V}_r\right|\left|d\vec{V}\right|\sin(\pi-\theta) = \left|\vec{V}_r\right|\left|d\vec{V}\right|\sin\theta' = \left|\vec{V}_r\right| h = -2 * Area
\end{aligned}
$$

where $Area = -\dfrac{\left|\vec{V}_r\right| h}{2}$. The negative sign is because of the definition of the area, which in this case is swept by $\vec{V}_r$ in the clockwise direction ($d\vec{V}$ points to the right side of $\vec{V}_r$).

$$
\begin{aligned}
\therefore SREH &= -\int_{0km}^{3km} \hat{k}\cdot\left[\vec{V}_r \times d\vec{V}\right] = \int_{0km}^{3km} d(SREH) \\
&= -2\times Total\_Area\_swepted\_by\_\vec{V}_r\_in\_3km\_depth
\end{aligned}
$$

b). If the storm-relative velocity at 0 and 3 km levels are $(u_{r1}, v_{r1})\, and\ (u_{r2}, v_{r2})$, respectively, show that SREH can be calculated as

$$SERH = u_{r2}v_{r1} - u_{r1}v_{r2}\,.$$

Hint: Think of how you calculated the storm-relative helicity in Problem 1 for each of those 6 layers. Also, $d\vec{V} = d\vec{V}_r$ because the storm motion vector is constant with height.

The SERH in the 3 km layer is:

$$
\begin{aligned}
SREH &= [(\vec{V}-\vec{C})\cdot\vec{\omega}_H]dz = (\bar{u}\hat{i} + \bar{v}\hat{j})\cdot\left[-\frac{dv}{dz}\hat{i} + \frac{du}{dz}\hat{j}\right]dz = (\bar{u}\hat{i} + \bar{v}\hat{j})\cdot(-dv\hat{i} + du\hat{j}) \\
&= -\frac{u_1+u_2}{2}(v_2 - v_1) + \frac{v_1+v_2}{2}(u_2 - u_1) \\
&= -\frac{1}{2}\left[u_1v_2 - u_1v_1 + u_2v_2 - u_2v_1 - v_1u_2 + v_1u_1 - v_2u_2 + v_2u_1\right] \\
&= u_2v_1 - u_1v_2
\end{aligned}
$$

c). Verify that for the hodograph shown below, and a zero storm-motion vector, that the above two methods for computing the SREH give the same results.

Solution: Just plug in the numbers.

d). Explain why larger SERH tends to promote longer lasting supercell storms.

See notes. The key is that it leads to large correlation between w’ and ζ’, implying large vorticity in updraft – from our analysis on pressure perturbation associated with rotation updraft, we understand rotation in updraft produces additional positive lifting therefore stronger updraft therefore stronger storms.

2. Wind Hodographs. A vertical wind profile of the horizontal is given in the following table:

z (height, km)	θ (direction, deg)	V(speed, m/s)
0	110	6
1	150	10
2	180	15
3	190	17
4	250	25
5	270	30
6	310	40

Assume that the storm motion vector is 225 degrees (from SW) at 12 m/s.

a). Plot the hodograph and the storm-relative velocity vectors at each level.

b). Calculate the horizontal vorticity (vector, in terms of the vorticity components or in magnitude and direction) in each of the six layers between the levels of observations.

c). Determine the mean (storm-relative) wind vector in each of the six layers.

d). Using the layer-mean wind obtained above, calculate the storm-relative environmental helicity in each of the six layers and determine the vertically integrated environmental helicity in the lowest three kilometers.

e). Discuss your results and their significance in terms of their effect on the behavior and type of storms that occur in such an environment

f). For this wind profile, what kind of CAPE values will give you a BRN that suggests a high probability of multicell and supercell storms, respectively?

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  Z(km)    Dir(deg)   V(m/s)     ua(m/s)    va(m/s)    ur(m/s)  vr(i)
  0.000    110.000     6.000     -5.638      2.052    -14.123    -6.433
  1.000    150.000    10.000     -5.000      8.660    -13.485     0.175
  2.000    180.000    15.000      0.000     15.000     -8.485     6.515
  3.000    190.000    17.000      2.952     16.742     -5.533     8.256
  4.000    250.000    25.000     23.492      8.551     15.007     0.065
  5.000    270.000    30.000     30.000      0.000     21.515    -8.485
  6.000    310.000    40.000     30.642    -25.711     22.157   -34.197

Layer omegax(1/s) omegay(1/s)  um(m/s)  vm(m/s)    H(m/s**2)  RH      OmegaS
   1  -0.66E-02   0.64E-03   -13.804    -3.129     0.089     0.949   0.63E-02
   2  -0.63E-02   0.50E-02   -10.985     3.345     0.086     0.932   0.75E-02
   3  -0.17E-02   0.30E-02    -7.009     7.386     0.034     0.975   0.33E-02
   4   0.82E-02   0.21E-01     4.737     4.161     0.124     0.891   0.20E-01
   5   0.86E-02   0.65E-02    18.261    -4.210     0.129     0.639   0.69E-02
   6   0.26E-01   0.64E-03    21.836   -21.341     0.548     0.697   0.18E-01

3km integrated helicity (m**2/s**2)=   209.603
BRN shear S (m/s) =    16.326
CAPE for BRN=10 is   1332.728
CAPE for BRN=45 is   5997.276
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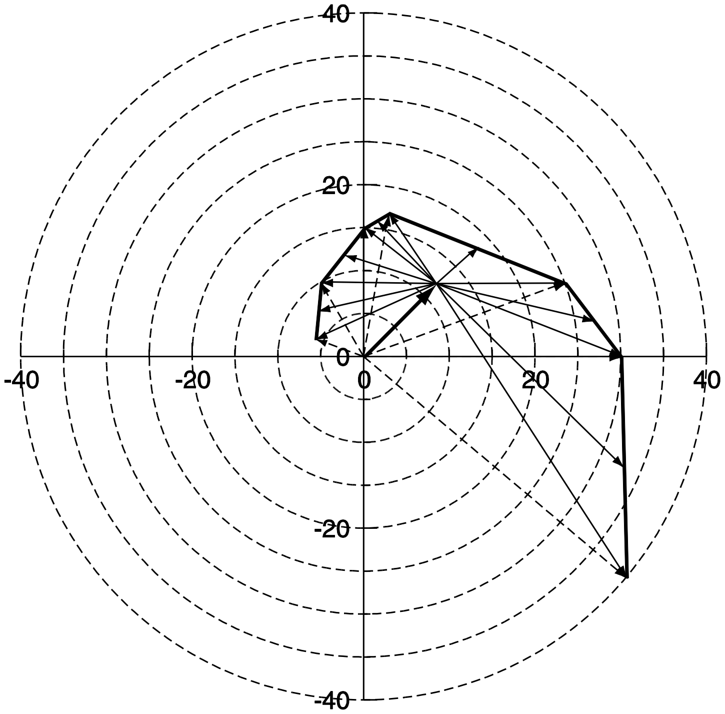
Hodograph:



g). For this wind profile, what kind of CAPE values will give you a BRN that suggests a high probability of multicell and supercell storms, respectively?

Multicell storms occur mostly when bulk Richardson number Rn = 2* CAPE/ $(V_{6km}-V_{BL})^2 > 45$.